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# Dynamical breaking of supersymmetry (II)*

He-Shan Song† and Guang-Nan Xu‡

† Center of Theoretical Physics, CCAST (World Laboratory), PO Box 8730, Beijing 100080, People's Republic of China and Department of Physics, Dalian University of Technology, Dalian 116024, People's Republic of China§

‡ Department of Physics, Liaoning Normal University, Dalian 116021, People's Republic of China

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**Abstract.** The supersymmetry breaking is discussed starting from the Wess–Zumino model without adding the Feyet–Iliopoulos term. It is shown, in terms of the Nambu–Jona-Lasinio mechanism, that the supersymmetry breaking can be realized dynamically even in the Wess–Zumino model.

## 1. Introduction

Some time ago the spontaneous breaking of supersymmetry was studied by many authors [1–5]. The usual method for realizing the supersymmetry breaking is to add a gauge invariant but parity-violating term [3] $\xi D$ (Feyet–Iliopoulos term) to the Lagrangian. The adding of the Feyet–Iliopoulos term to the Lagrangian leads to mass splitting between bosons and fermions and hence the supersymmetry should be broken down spontaneously. Feyet and Iliopoulos have pointed out that a spontaneous breaking of supersymmetry does not occur for the interaction of a chiral scalar superfield with itself [3, 5].

It is shown in our previous work [6], however, that the supersymmetry can break down dynamically without adding the parity-violating term. We have shown, in terms of the Nambu–Jona-Lasinio mechanism [7], that the supersymmetry breaking can be realized by the fermion pair condensation $\langle\bar{\psi}\psi\rangle$ even in the simplest supersymmetric theory of a single real supermultiplet [8].

The purpose of this present paper is to generalize our idea to the Wess–Zumino model [9] and show, in terms of Nambu–Jona-Lasinio mechanism, that the supersymmetry can be realized dynamically even in the Wess–Zumino model. We will show that the nonlinear interaction between the fields will generate different dynamical masses for different fields and hence leads to the supersymmetry breaking.

The paper is organized as follows. In section 2 we review briefly the Nambu–Jona-Lasinio mechanism. In section 3 we will establish self-consistency equation for order parameter starting from a massless Wess–Zumino model and show that the scalar field of the theory has a non-vanishing vacuum expectation value due to the contribution from the self-energy of the fermion that leads to the mass splitting between bosons and fermions. The conclusions are summarized at the end of the paper.

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§ Mailing address.

## 2. The Nambu–Jona-Lasinio mechanism

Nambu and Jona-Lasinio have suggested [7], in 1961, that the nucleon mass arises largely as a self-energy of some primary massless fermion fields and real nucleons are regarded as quasi-particle excitations. The Nambu–Jona-Lasinio model is based on the Lagrangian density

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\mathrm{I}} \tag{1}$$

where $\mathcal{L}_0$ is the free Lagrangian

$$\mathcal{L}_0 = \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi \tag{2}$$

and $\mathcal{L}_{\mathrm{I}}$ is a four-fermions interaction of the type

$$\mathcal{L}_{\mathrm{I}} = \tfrac{1}{2} g[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]. \tag{3}$$

Instead of diagonalizing $\mathcal{L}_0$ and treating $\mathcal{L}_{\mathrm{I}}$ as perturbation, Nambu and Jona-Lasinio have introduced a self-energy term

$$\mathcal{L}_{\mathrm{s}} = \delta m \bar{\psi}\psi \tag{4}$$

to the Lagrangian and rewrite the Lagrangian (1) as

$$\mathcal{L} = (\mathcal{L}_0 + \mathcal{L}_{\mathrm{s}}) + (\mathcal{L}_{\mathrm{I}} - \mathcal{L}_{\mathrm{s}}). \tag{5}$$

The crucial assumption of the Nambu–Jona-Lasinio model is that despite the vanishing of the bare fermion mass, the physical mass $m$ of the fermion is non-zero. Then by using Dyson's mass renormalization prescription, the Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_0' + \mathcal{L}_{\mathrm{I}}' \tag{6}$$

with

$$\mathcal{L}_0' = \bar{\psi}(\mathrm{i}\partial\!\!\!/ - \delta m)\psi \tag{7}$$

$$\mathcal{L}_{\mathrm{I}}' = \mathcal{L}_{\mathrm{I}} + \delta m \bar{\psi}\psi. \tag{8}$$

The self-mass $\delta m$ is given by

$$\delta m = \Sigma^*(P)|_{P=m} \tag{9}$$

and the right-hand side of (9) can be calculated, to the first order in $g$, from the self-energy diagram of the fermion. Since $m_0 = 0$, we have $m = \delta m$ and so we get the self-consistent equation for the physical mass of fermion:

$$m = \Sigma^*(P)|_{P=m}. \tag{10}$$

Nambu and Jona-Lasinio have evaluated that

$$m \simeq 2\mathrm{i}g \,\mathrm{Tr}\, S_{\mathrm{F}}(0) \tag{11}$$

where

$$S_F(0) = -i\langle\psi(x)\bar{\psi}(x)\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{1}{\not{P} - m} \tag{12}$$

represents the contribution of closed-loop (self-energy) diagram. Equation (11) gives

$$\frac{m^2}{\Lambda^2}\ln\left(\frac{\Lambda^2}{m^2}+1\right) = 1 - \frac{2\pi^2}{g\Lambda^2} \tag{13}$$

by introducing invariant momentum cut-off $P^2 = \Lambda^2$. Since the right-hand side of (13) is positive and $\leqslant 1$ for real $\Lambda$ and $m$, we get a constraint equation for $g$ and $\Lambda$:

$$0 < \frac{2\pi^2}{g\Lambda^2} < 1. \tag{14}$$

We see that in the Nambu–Jona-Lasinio theory starting from zero-mass fermion one generates the dynamical mass of fermion self-consistently.

Lurie and Macfarlane have shown [10] the equivalence between Lagrangian field theory of four-fermion type considered by Nambu and Jona-Lasinio and a Lagrangian theory of the same fermion fields with coupling of Yukawa type

$$\mathcal{L}_Y = \mathcal{L}_0 + G\bar{\psi}\psi\Phi_s + G\bar{\psi}\gamma_5\Phi_P \tag{15}$$

and obtained the physical fermion mass in the equivalent Yukawa theory in the same way.

## 3. Dynamical breaking of supersymmetry

Many authors have discussed [11–13] dynamical breaking of chiral symmetry in terms of the Nambu–Jona-Lasinio mechanism. Most recently, the Nambu–Jona-Lasinio mechanism has become popular as a toy model to investigate various aspects of low-energy hadron dynamics.

The aim of this section is to start from a massless Wess–Zumino model (which involves Yukawa type interaction in its Lagrangian, in terms of the Nambu–Jona-Lasinio mechanism mentioned above), and to show that the contribution of the self-energy part of fermion to the self-consistency equation for the order parameter—see (50)—leads to a non-vanishing vacuum expectation value of the scalar field and provides different masses to the bosons and fermions in the supermultiplet. The masses of the fields will be determined in terms of coupling constant $g$ and momentum cut-off $\Lambda$ and so the masses are not fixed parameters but rather dynamical quantities [7].

Now let us consider the massless Wess–Zumino model of the supersymmetry described by the Lagrangian density [14, 15]

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_g \tag{16}$$

with

$$\mathcal{L}_0 = \tfrac{1}{2}[(\partial_\mu A)^2 + (\partial_\mu B)^2 + i\bar{\psi}\not{\partial}\psi + F^2 + G^2] \tag{17}$$

$$\mathcal{L}_g = -g[(A^2 - B^2)F + 2ABG + \bar{\psi}(A - \gamma_5 B)\psi]. \tag{18}$$

Here $A$ and $B$ are respectively a scalar and a pseudoscalar field, $\psi$ is a Majorana spinor and $F$ and $G$ are auxiliary fields.

Using the Euler–Lagrange formalism, we find following equations of motion for the auxiliary fields:

$$F = g(A^2 - B^2) \tag{19}$$

$$G = 2gAB \tag{20}$$

which can be used to eliminate the auxiliary fields from the Lagrangian (16). The result of their elimination is

$$\mathcal{L} = \tfrac{1}{2}[(\partial_\mu A)^2 + (\partial_\mu B)^2 + \mathrm{i}\bar{\psi}\partial\!\!\!/\psi] - \tfrac{1}{2}g^2(A^2 + B^2)^2 - g\bar{\psi}(A - \gamma_5 B)\psi. \tag{21}$$

The Lagrangian (21) is invariant under the supertransformations

$$\begin{aligned} \delta A &= \bar{\xi}\psi \\ \delta B &= \bar{\xi}\gamma_5\psi \\ \delta\psi &= -[\mathrm{i}\partial\!\!\!/ + g(A + \gamma_5 B)](A + \gamma_5 B)\xi. \end{aligned} \tag{22}$$

The equations of motion following from the Lagrangian (21) are given by

$$\Box A + 2g^2 A(A^2 + B^2) = -g\bar{\psi}\psi \tag{23}$$

$$\Box B + 2g^2 B(A^2 + B^2) = g\bar{\psi}\gamma_5\psi \tag{24}$$

$$[\mathrm{i}\partial\!\!\!/ - 2g(A - \gamma_5 B)]\psi = 0. \tag{25}$$

Now we introduce external sources $J_A(x)$, $J_B(x)$ and $J_\psi(x)$ coupled to the fields $A$, $B$ and $\psi$ respectively, and write the Lagrangian as

$$\mathcal{L}[J] = \mathcal{L} + J_A A + J_B B + \bar{J}_\psi\psi + \bar{\psi}J_\psi. \tag{26}$$

The equations of motion then turn out to be

$$\Box A + 2g^2 A(A^2 + B^2) = -g\bar{\psi}\psi + J_A(x) \tag{27}$$

$$\Box B + 2g^2 B(A^2 + B^2) = g\bar{\psi}\gamma_5\psi + J_B(x) \tag{28}$$

$$[\mathrm{i}\partial\!\!\!/ - 2g(A - \gamma_5 B)]\psi = -2J_\psi(x). \tag{29}$$

As usual, the generating functional is given by

$$Z[J] = \int [\mathrm{d}A][\mathrm{d}B][\mathrm{d}\psi][\mathrm{d}\bar{\psi}]\exp\left\{\frac{\mathrm{i}}{\hbar}\int \mathrm{d}^4x\, \mathcal{L}[J]\right\}. \tag{30}$$

By using the generating functional $Z[J]$ the vacuum expectation value of a field, say $A(x)$, in the presence of external sources can be found as

$$\langle A(x)\rangle_0^J = \frac{\delta W[J]}{\delta J_A(x)} = \frac{1}{Z[J]}\int [\mathrm{d}A][\mathrm{d}B][\mathrm{d}\psi][\mathrm{d}\bar{\psi}]\exp\left\{\frac{\mathrm{i}}{\hbar}\int \mathrm{d}^4x\, \mathcal{L}[J]\right\} \tag{31}$$

with

$$W[J] = (\hbar/\mathrm{i})\ln Z[J]. \tag{32}$$

Now let us take vacuum expectation values of (27) and (28), then one gets

$$\Box\langle A\rangle_0^J + 2g^2\langle A^3\rangle_0^J + 2g^2\langle AB^2\rangle_0^J = -g\langle\bar{\psi}\psi\rangle_0^J + J_A(x) \tag{33}$$

$$\Box\langle B\rangle_0^J + 2g^2\langle BA^2\rangle_0^J + 2g^2\langle B^3\rangle_0^J = g\langle\bar{\psi}\gamma_5\psi\rangle_0^J + J_B(x). \tag{34}$$

By using (31) the vacuum expectation values of the terms such as $\langle A^3\rangle_0^J$, $\langle AB^2\rangle_0^J$ in (33) and (34) are easy to calculate and the results are given in the forms

$$\langle A^3\rangle_0^J = (\langle A\rangle_0^J)^3 + 3\frac{\hbar}{\mathrm{i}}\langle A\rangle_0^J\frac{\delta\langle A\rangle_0^J}{\delta J_A(x)} + \left(\frac{\hbar}{\mathrm{i}}\right)^2\frac{\delta^2\langle A\rangle_0^J}{\delta J_A^2(x)}$$

$$\langle AB^2\rangle_0^J = \langle A\rangle_0^J\left[(\langle B\rangle_0^J)^2 + \frac{\hbar}{\mathrm{i}}\frac{\delta\langle B\rangle_0^J}{\delta J_B(x)}\right] + 2\frac{\hbar}{\mathrm{i}}\langle B\rangle_0^J\frac{\delta\langle A\rangle_0^J}{\delta J_B(x)} + \left(\frac{\hbar}{\mathrm{i}}\right)^2\frac{\delta^2\langle A\rangle_0^J}{\delta J_B^2(x)}$$

and similarly for the terms $B^3$ and $BA^2$. So in the lowest order approximation in $\hbar$, (33) and (34) can be reduced to

$$\Box\langle A\rangle_0^J + 2g^2(\langle A\rangle_0^J)^3 + 2g^2\langle A\rangle_0^J(\langle B\rangle_0^J)^2 = -g\langle\bar{\psi}\psi\rangle_0^J + J_A(x) \tag{35}$$

$$\Box\langle B\rangle_0^J + 2g^2\langle B\rangle_0^J(\langle A\rangle_0^J)^2 + 2g^2(\langle B\rangle_0^J)^3 = g\langle\bar{\psi}\gamma_5\psi\rangle_0^J + J_B(x) \tag{36}$$

or

$$2g^2A_0^3 + 2gA_0B_0^2 = -\langle\bar{\psi}\psi\rangle_0^J = \mathrm{i}\,\mathrm{Tr}\,S_\mathrm{F}(0) \tag{37}$$

$$2gB_0^3 + 2gB_0A_0^2 = \langle\bar{\psi}\gamma_5\psi\rangle = -\mathrm{i}\,\mathrm{Tr}[\gamma_5 S_\mathrm{F}(0)] \tag{38}$$

in the limit $J \to 0$. In (37) and (38),

$$A_0 = \langle A\rangle_0^J|_{J=0} \tag{39}$$

$$B_0 = \langle B\rangle_0^J|_{J=0} \tag{40}$$

and

$$\mathrm{Tr}\,S_\mathrm{F}(0) = \mathrm{i}\langle\bar{\psi}(x)\psi(x)\rangle_0^J|_{J=0} \tag{41}$$

$$\mathrm{Tr}[\gamma_5 S_\mathrm{F}(0)] = \mathrm{i}\langle\bar{\psi}(x)\gamma_5\psi(x)\rangle_0^J|_{J=0}. \tag{42}$$

Equations (41) and (42) represent closed-loop diagrams which correspond to the self-energy of the fermion. The self-energy of the fermion can be evaluated from (29). From (29) we obtain

$$\begin{aligned}\mathrm{i}\partial\!\!\!/ S_\mathrm{F}^J(x,y) = \delta^4(x-y) &- 2\mathrm{i}g\langle TA(x)\psi(x)\bar{\psi}(y)\rangle_0^J \\ &+ 2\mathrm{i}g\langle TB(x)\gamma_5\psi(x)\bar{\psi}(y)\rangle_0^J + 2\mathrm{i}J_\psi(x)\langle\bar{\psi}(y)\rangle_0^J.\end{aligned} \tag{43}$$

By using the general relation

$$-\mathrm{i}\langle T\varphi(x')\psi(x)\bar{\psi}(y)\rangle_0^J = \frac{\hbar}{\mathrm{i}}\frac{\delta S_{\mathrm{F}}^J(x,y)}{\delta J_\varphi(x')} + S_{\mathrm{F}}^J(x,y)\langle\varphi(x')\rangle_0^J$$

equation (43) reduces to the Schwingers functional differential equation for $S_{\mathrm{F}}^J(x,y)$:

$$\begin{aligned} \mathrm{i}\partial\!\!\!/ S_{\mathrm{F}}^J(x,y) &= \delta^4(x-y) + 2g\left[\langle A\rangle_0^J S_{\mathrm{F}}^J(x,y) + \frac{\hbar}{\mathrm{i}}\frac{\delta S_{\mathrm{F}}^J(x,y)}{\delta J_A(x)}\right] \\ &\quad - 2g\left[\langle B\rangle_0^J \gamma_5 S_{\mathrm{F}}^J(x,y) + \frac{\hbar}{\mathrm{i}}\gamma_5\frac{\delta S_{\mathrm{F}}^J(x,y)}{\delta J_B(x)}\right] + 2\mathrm{i}J_\psi(x)\langle\bar{\psi}(y)\rangle_0^J. \end{aligned} \tag{44}$$

In the lowest order approximation in $\hbar$ and in the limit $J \to 0$, the equation (44) turns out to be

$$[\mathrm{i}\partial\!\!\!/ - 2g(A_0 - \gamma_5 B_0)]S_{\mathrm{F}}(x-y) = \delta^4(x-y). \tag{45}$$

So we obtain, in the momentum space, the fermion propagator

$$S_{\mathrm{F}}(P) = \frac{1}{P\!\!\!/ - 2g(A_0 - \gamma_5 B_0)}. \tag{46}$$

Substituting the fermion propagator into (37) and (38) gives

$$2gA_0(A_0^2 + B_0^2) = \mathrm{Tr}\int \frac{\mathrm{d}^4 P}{(2\pi)^4}\frac{\mathrm{i}}{P\!\!\!/ - 2g(A_0 - \gamma_5 B_0)} \tag{47}$$

$$2gB_0(A_0^2 + B_0^2) = -\mathrm{Tr}\int \frac{\mathrm{d}^4 P}{(2\pi)^4}\frac{\mathrm{i}\gamma_5}{P\!\!\!/ - 2g(A_0 - \gamma_5 B_0)}. \tag{48}$$

A straightforward calculation shows that (47) and (48) can be simplified to the same equation

$$A_0^2 + B_0^2 = 4\int \frac{\mathrm{d}^4 P}{(2\pi)^4}\frac{\mathrm{i}}{P^2 - 4g^2(A_0^2 + B_0^2)}. \tag{49}$$

We see from (49) that the vacuum expectation values of the scalar field $A$ and pseudoscalar field $B$ are completely determined by the self-energy of the fermion. Note that (49) determines only the magnitude of $C_0 = (A_0, B_0)$, not its direction. The direction is essentially arbitrary. The arbitrariness in the direction of $C_0$ reflects infinite degeneracy of the vacuum state. If we take $C_0$ to lie along a particular direction, say $A_0$ direction, then (49) becomes simply an equation for $A_0$:

$$A_0^2 = 4\int \frac{\mathrm{d}^4 P}{(2\pi)^4}\frac{\mathrm{i}}{P^2 - 4g^2 A_0^2}. \tag{50}$$

The frame in which $B_0 = 0$ is characterized by a vacuum state which is an eigenstate of parity. For the space inversion invariance to hold (the Lagrangian (21) is invariant under the space inversion transformation), here and afterwards we take $B_0 = 0$. Equation (50) is a self-consistency equation for order parameter $A_0$. In (50), if we set

$$m_\psi^2 = 4g^2 A_0^2 \tag{51}$$

then (50) reduces to the self-consistency equation for the physical fermion mass $m_\psi$:

$$m_\psi^2 = 16g^2 \int \frac{d^4P}{(2\pi)^4} \frac{i}{P^2 - m_\psi^2}. \tag{52}$$

The integration in (52) is quadratically divergent. Introducing a momentum cut-off $\Lambda$ and performing Wick rotation one can make the integration meaningful. The result is given by

$$m_\psi^2 = \frac{g^2}{\pi^2}\left[\Lambda^2 - m_\psi^2 \ln\frac{\Lambda^2}{m_\psi^2}\right] \tag{53}$$

or

$$A_0^2 = \frac{\Lambda^2}{4\pi^2}\left[1 - \frac{4g^2A_0^2}{\Lambda^2}\ln\frac{\Lambda^2}{4g^2A_0^2}\right]. \tag{54}$$

Translating the scalar field $A$ as

$$A \longrightarrow A' = A - A_0 \tag{55}$$

and rewriting the Lagrangian (21) in terms of $A + A_0$ instead of $A$, gives

$$\begin{aligned}\mathcal{L} = \tfrac{1}{2}[(\partial_\mu A)^2 + (\partial_\mu B)^2 + i\bar\psi\partial\!\!\!/\psi - m_A^2A^2 - m_B^2B^2 - m_\psi\bar\psi\psi] \\ - \tfrac{1}{2}g^2(A^2+B^2) - g\bar\psi(A-\gamma_5 B)\psi - 2g^2A_0A(A_0^2 + A^2 + B^2)\end{aligned} \tag{56}$$

with

$$m_A^2 = 6g^2A_0^2 \tag{57}$$

$$m_B^2 = 2g^2A_0^2 \tag{58}$$

$$m_\psi = 2gA_0. \tag{59}$$

Evidently, the boson fields $A$, $B$ and the fermion field $\psi$ have acquired different masses $m_A$, $m_B$ and $m_\psi$, respectively, due to the non-vanishing vacuum expectation value of the field $A$ which is determined by the self-energy of the fermion. So the supersymmetry is broken dynamically.

If we start from massive Wess–Zumino model, the masses of bosons and fermions after the supersymmetry breakdown are given, respectively, by

$$m_A^2 = m_0^2 + 6gm_0A_0 + 6g^2A_0^2 \tag{60}$$

$$m_B^2 = m_0^2 + 2gm_0A_0 + 2g^2A_0^2 \tag{61}$$

$$m_\psi^2 = m_0^2 + 4gm_0A_0 + 4g^2A_0^2 \tag{62}$$

where $m_0$ is the original mass of bosons and fermions.

From (19) and (20) we see that the vacuum expectation value of the auxiliary fields

$$\langle F\rangle = g\langle A^2\rangle \approx gA_0^2 \neq 0 \tag{63}$$

$$\langle G\rangle = 2g\langle A\rangle\langle B\rangle = 0. \tag{64}$$

The only case where $\langle F\rangle = \langle G\rangle = 0$ is $A_0 = B_0 = 0$. This is another evidence of the supersymmetry breaking.

It should be noted that in $QCD$, the vacuum expectation value of $\bar{\psi}\psi$ serves as an order parameter for spontaneously broken chiral symmetry [16, 17]. It is therefore plausible to suppose [18] that fermion bilinears have vacuum expectation values in supersymmetric theories. In supersymmetry theory with chiral multiplets a non-zero vacuum expectation value of $\bar{\psi}\psi$ would break supersymmetry as well as chiral symmetry.

Note that, in our model, there are no Goldstone boson or fermion after the supersymmetry breakdown. This result is very similar to the symmetry breaking in linear $\sigma$-model [19]. In the linear $\sigma$-model the linear term $C\sigma$ induces symmetry breaking (externally) but there is no Goldstone boson in the linear $\sigma$-model.

Girardello *et al* [20] have also claimed that there are no Goldstone fermions associated with supersymmetry breaking.

## 4. Conclusions and discussions

The dynamical breaking of supersymmetry is studied in terms of the Nambu–Jona-Lasinio mechanism. Starting from a massless Wess–Zumino model we have shown that the nonlinear interaction between the fields generates different dynamical masses for different fields and hence leads to the supersymmetry breaking.

The supersymmetry breaking and its behaviour at finite temperature remains a topic of considerable interest. As shown in this work, in the theories involving Yukawa coupling, the zero-point energy of fermion pair can move the minimum of the effective potential away from zero so as to break symmetry and generate masses. This fact suggests that if we can establish a self-consistency equation at finite temperature then one can investigate the supersymmetry behaviour at finite temperature by solving the self-consistency equation at finite temperature. This is particularly interesting for us. Since the supersymmetry is a beautiful symmetry, one would like somehow to maintain it at high temperature. Recent work of Cabo *et al* [21] shows that invariance of the thermal average in supersymmetric theory should be retained by the Wess–Zumino model.

It is worth noticing that the models we have considered do not involve gauge multiplet in its Lagrangian. For the case which involves gauge multiplet, we will deal with in our forthcoming work and we would like to see what happens.

## References


[1] Iliopoulos J and Zumino B 1974 *Nucl. Phys.* B **76** 310
[2] Salam A and Strasdee J 1974 *Phys. Lett.* **49B** 465
[3] Feyet P and Iliopoulos J 1974 *Phys. Lett.* **51B** 461
[4] Witten E 1981 *Phys. Lett.* **105B** 267
[5] O'Raifeartigh L 1975 *Nucl. Phys.* B **96** 331
[6] Guang-Nan Xu and He-Shan Song 1992 *J. Phys. A: Math. Gen.* **25** 4941
[7] Nambu Y and Jona-Lasinio G 1961 *Phys. Rev.* **122** 345

[8] Paranjape M B, Taormina A and Wijewardhana L C R 1983 *Phys. Rev. Lett.* **50** 1350
[9] Wess J and Zumino B 1971 *Phys. Lett.* **49B** 52
[10] Lurie P and Macfarlane A J 1964 *Phys. Rev.* B **136** 816
[11] Finger J R and Mandula J E 1982 *Nucl. Phys.* B **199** 1
[12] Bernard V 1986 *Phys. Rev.* D **34** 1601
[13] Bernard V and Meißner U G 1991 *Phys. Lett.* B **266** 403
[14] Sohnius M F 1986 *Supersymmetry and Supergravity* (A reprint volume of *Phys. Rep.*) ed M Jakob
[15] Boyanovsky D 1984 *Phys. Rev.* D **29** 743
[16] Ryang S and He-Shan Song 1984 *Phys. Lett.* **154B** 310
[17] He-Shan Song, An Y and Li X Q 1989 *Commun. Theor. Phys.* **11** 299
[18] Witten E 1981 *Nucl. Phys.* B **188** 513
[19] Lee B W 1969 *Nucl. Phys.* B **9** 649
[20] Giradello L, Grisaru M T and Salomonson P 1981 *Nucl. Phys.* B **178** 331
[21] Cabo A, Chaichian M and Shabad A E *Preprint* CERN-TH 6589/92